Bachelor of Science (B.Sc.) Semester—II
(C.B.S.) Examination
MATHEMATICS
Compulsory Paper—I
(M_3—Geometry, Differential and Difference Equations)

Time—Three Hours] [Maximum Marks—60

N.B. :— (1) Solve all the **FIVE** questions.
(2) All questions carry equal marks.
(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT—I

1. (a) Obtain the equation of sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$ as the great circle. 6

(b) Find the two tangent planes to the sphere
$x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$
which are parallel to the plane
$2x + 2y = z$. 6

OR

1. (c) Find the equation of the right circular cone which passes through the point (1, 1, 2) and has its vertex at the origin and axis $\frac{x}{2} = \frac{-y}{4} = \frac{z}{3}$. 6

(d) Find the equation of the right circular cylinder of radius 2, whose axis passes through (1, 0, 3) and has direction cosines proportional to (2, 3, 1). 6

UNIT—II

2. (a) Show that the equation

$$(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$$

is exact and hence solve it. 6

(b) Solve the differential equation

$$(1 - x^2)\frac{dy}{dx} + 2xy = x(1 - x^2)^{1/2}.$$ 6

OR

2. (c) Solve the differential equation

$$(x^2 + y^2)dx + xydy = 0$$

by finding integrating factor. 6

(d) Solve the equation

$$p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0,$$

where $p = \frac{dy}{dx}$. 6

UNIT—III

3. (a) Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x}$. 6

(b) Solve $(D^2 - 5D + 6)y = \cos 2x$,

where $D \equiv \frac{d}{dx}$. 6

OR

3. (c) Solve $(x^2D^2 - xD + 4)y = \cos(\log x)$. 6

(d) Solve $y^{(2)} + y = \sec x$

by the method of variation of parameters. 6

UNIT—IV

4. (a) From the relation $u_x = c_1 3^x + c_2(-1)^x$, derive a difference equation by eliminating the arbitrary constants c_1 and c_2. 6

(b) Solve :

$$u_{x+2} - 7u_{x+1} + 10\, u_x = 12(u)^x.$$ 6

OR

4. (c) Solve :

$$u_{x+2} + u_{x+1} + u_x = x^2 + x + 1.$$ 6

(d) Solve :

$$u(x + 1) - 2u(x) = \cos nx.$$ 6

UNIT—V

5. (a) Show that the two spheres

$x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$ and

$x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$

are orthogonal. 1½

(b) Define the right circular cone. 1½

(c) Reduce the differential equation

$$\frac{dy}{dx} - \frac{1}{x}y = -x^2y^2$$ to the linear form. 1½

(d) Solve :

$$p = \log(px - y), \text{ where } p = \frac{dy}{dx}.$$ 1½

(e) Solve :

$$(D^2 + 1)y = 0, \text{ where } D \equiv \frac{d}{dx}.$$ 1½

(f) Find the particular integral of

$(D^2 - 4)y = e^{2x}$. 1½

(g) Solve :

$u_{x+2} - 4u_x = 0$. 1½

(h) Define the difference equation. Find the order of the difference equation

$u_{x+3} - 6u_{x+2} + 5u_{x+1} - 2u_x = 0$. 1½